

Lesson Plan

NEUROHEADWAY

Neuroscience Into Education

Lesson Plan - Final Payment

The quantity of work is for two 45- minutes lessons- one on the topic by using the Power point presentation, and the second lesson is for consolidation by working on the worksheet.

Pre-knowledge

- Logarithms- definition, laws and properties
- Substitution into a formula
- Solving algebraic equations
- All Financial Mathematics formulae

Aims

- To learn the terminology in Mathematics of Finance
- Ability to use all Financial Mathematics formulae and to choose the right formula for a certain situation
- Ability to understand and discuss the meaning of the final answer
- Logical and critical thinking

Resources

- Power Point
- Worksheet

Activities

- By following the Power Point presentation, the learners need to learn how to apply the basic formulae and do the examples together.
- In groups or individually they must solve the application problems in the worksheet.

Worksheet

The worksheet is divided into three sections:

Section A- easy questions for the lower ability pupils

Section B- middle level of difficulty question

Section C- advanced (difficult) question for the gifted and talented pupils.

Note: The learners are encouraged to use a calculator only once in their solution in Mathematics of Finance. Only the final answer should be rounded off.

Power point Example

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EXAMPLE 1

A family inherited £100 000. They invested the money in a bank which pays an interest of 5% per annum, compounded quarterly. The family would like to withdraw £3 000 at the end of each quarter.

- (a) After how many quarters will the money in the bank account be finish?
- (b) What is the final amount that they will withdraw from the bank account?

Solution

$$(a) \quad P = x \cdot \frac{1-(1+i)^{-n}}{i}$$

$$100\,000 = 3\,000 \cdot \frac{1-(1+\frac{0.05}{4})^{-n}}{\frac{0.05}{4}}$$

$$\frac{100\,000 \cdot \frac{0.05}{4}}{3\,000} = 1 - (1 + \frac{0.05}{4})^{-n}$$

$$\frac{5}{12} = 1 - (1 + \frac{0.05}{4})^{-n}$$

$$(1 + \frac{0.05}{4})^{-n} = 1 - \frac{5}{12}$$

$$(1 + \frac{0.05}{4})^{-n} = \frac{7}{12}$$

$$\log(1 + \frac{0.05}{4})^{-n} = \log \frac{7}{12}$$

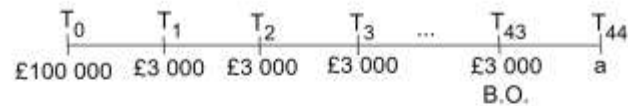
$$-n \cdot \log(1 + \frac{0.05}{4}) = \log \frac{7}{12}$$

$$n = -\frac{\log \frac{7}{12}}{\log(1 + \frac{0.05}{4})}$$

$$n \approx 43.39$$

$$n = 44 \text{ quarters}$$

(b)



$$i = 5\% \text{ p.a.c.q.}$$

The last payment $a < 30\,000$.

To calculate the value of a we need the balance outstanding at T_{43} .

Then $a = B.O. (1 + i)$ is the balance outstanding with the interest for one quarter.

$$B.O. = F_{\text{loan}} - F_{\text{payments}}$$

$$B.O. = P \cdot (1 + i)^n - x \cdot \frac{[(1 + i)^n - 1]}{i}$$

$$B.O. = 100\,000 \cdot \left(1 + \frac{0.05}{4}\right)^{43} - 3\,000 \cdot \frac{\left[\left(1 + \frac{0.05}{4}\right)^{43} - 1\right]}{\frac{0.05}{4}}$$

$$B.O. \approx £1\,155.96$$

$$a = 1\,155.96 \left(1 + \frac{0.05}{4}\right)$$

$a \approx £1\,170.41$ is the final amount that they will receive at the end of the 44th quarter

EXAMPLE 2

Mr Stan takes a loan of £400 000 from a bank for 30 years to start a new business. The bank charges interest of 6% per annum, compounded monthly. Mr Stan will repay the loan with equal instalments at the end of each month.

- Calculate the amount of the monthly payments.
- If Mr Stan can afford to pay £4 000 every month instead, how soon will he pay off the loan?
- How much will the last payment be?
- If Mr Stan pays £4 000 at the end of every month instead of the original monthly payments, how much will he save in total for the period of payment?

Solution

Note: The loan is always a present value.

$$(a) \quad P = x \cdot \frac{1-(1+i)^{-n}}{i} \quad \text{where } n = 30 \times 12 = 360 \text{ months}$$

$$400\,000 = x \cdot \frac{1-(1+\frac{0.06}{12})^{-360}}{\frac{0.06}{12}}$$

$$x = \frac{400\,000 \cdot \frac{0.06}{12}}{1-(1+\frac{0.06}{12})^{-360}}$$

$$x \approx \text{£ } 2\,398.20 \text{ per month}$$

$$(b) \quad P = x \cdot \frac{1-(1+i)^{-n}}{i}$$

$$400\,000 = 4\,000 \cdot \frac{1-(1+\frac{0.06}{12})^{-n}}{\frac{0.06}{12}}$$

$$\frac{400\,000 \cdot \frac{0.06}{12}}{4\,000} = 1 - (1 + \frac{0.06}{12})^{-n}$$

$$0.5 = 1 - (1 + \frac{0.06}{12})^{-n}$$

$$(1 + \frac{0.06}{12})^{-n} = 1 - 0.5$$

$$(1 + \frac{0.06}{12})^{-n} = 0.5$$

$$\log(1 + \frac{0.06}{12})^{-n} = \log(0.5)$$

$$-n \cdot \log(1 + \frac{0.06}{12}) = \log(0.5)$$

$$n = -\frac{\log(0.5)}{\log(1 + \frac{0.06}{12})}$$

$$n \approx 138.98$$

$$n = \mathbf{139 \text{ months}}$$

Teachers Evaluation and Handouts

Final Payment - Worksheet

Section A.

1. Anthony is planning to start a small business. He takes out a loan of £200,000 from a bank. The bank charges him interest at 7% per annum, compounded quarterly. Anthony can afford to pay £8 000 at the end of each quarter.
 - (a) How long (in years and months) will it take him to repay this loan to the bank?
 - (b) Calculate the value of the last quarterly payment.
2. A student takes a study loan of £2,500 from a bank, which charges interest at 4.5% per annum, compounded monthly. He plans to repay the loan by means of equal payments of £200 at the end of each month. His payments will start four years after the loan is granted, that is when he has graduated.
 - (a) How much will the loan be worth after 4 years, together with the interest?
 - (b) Calculate how many months it will take for the student to fully repay the loan.
 - (c) Calculate the value of the last payment, which will be less than £200.
3. For the last 35 years, Savana saved £500 at the end of each month into a pension fund, which pays interest at 4% per annum, compounded monthly. Savana has retired now and would like to withdraw £2,000 at the end of each month.
 - (a) Calculate the number of months it will be before the pension fund is depleted.
 - (b) Calculate the value of her last withdrawal.

Section B.

1. Lynn has just started work and buys a car by borrowing £12,000 from a bank. The bank offers an interest rate of 6% per annum, compounded monthly for 5 years. She will pay the bank in equal instalments at the end of each month. Lynn decides to pay an extra £100 per monthly payment.
 - (a) What is the monthly payment required by the bank, and how much does she decide to pay each month?
 - (b) Calculate how many payments with the extra £100 will she make to pay off the loan?
 - (c) Calculate the value of the last payment.
 - (d) How much will she save by paying the extra £100 per month?

Section C.

1. A man buys a house for £220,000. He pays a deposit of 15% and he takes a loan from a bank for the remaining amount. He will repay the loan by the means of equal instalments at the end of each month for 25 years. The bank charges interest of 4.8% per annum, compounded monthly.

- (a) Calculate the value of the loan.
- (b) Calculate the value of the monthly instalments.
- (c) If he can afford to pay £2,000 per month, calculate how many months it will take him to repay the loan?
- (d) Calculate the value of the last payment.
- (e) How much will he save by paying £2,000 per month?
- (f) If he pays £2,000 per month, calculate how much interest he will pay the bank by the time the loan is amortised.