

Mathematical Analysis – SERIES

AIMS

Grasp an intuition of some of the objects studied in mathematical analysis, such as the *series*.

RESOURCES

PowerPoint, paper, pen, coloured pencils, a notch of vibrant creativity, and some vivid intuition.

Each activity needs to be properly introduced by a detailed discussion, and perhaps guided to favour the intuition of some of the students. Each activity takes at least 15 mins, and time for additional discussions has to be taken into account.

In any case, the time needed for each activity may considerably vary according to the interests of the class and the specific skills of the students.

The material and the activities presented in this tutorial are extremely challenging, so it is important to follow the students, to stimulate and encourage them, and no one has to be discouraged if (s)he does not grasp the ideas at once.

SERIES

Geometric series, filling the square with squares.

Other geometric methods.

Cancellations.

Series

A *series* is an “infinite sum”.

When we sum up infinite numbers, we may obtain “infinity” as a result, for instance

$$1 + 1 + 1 + 1 + \dots$$

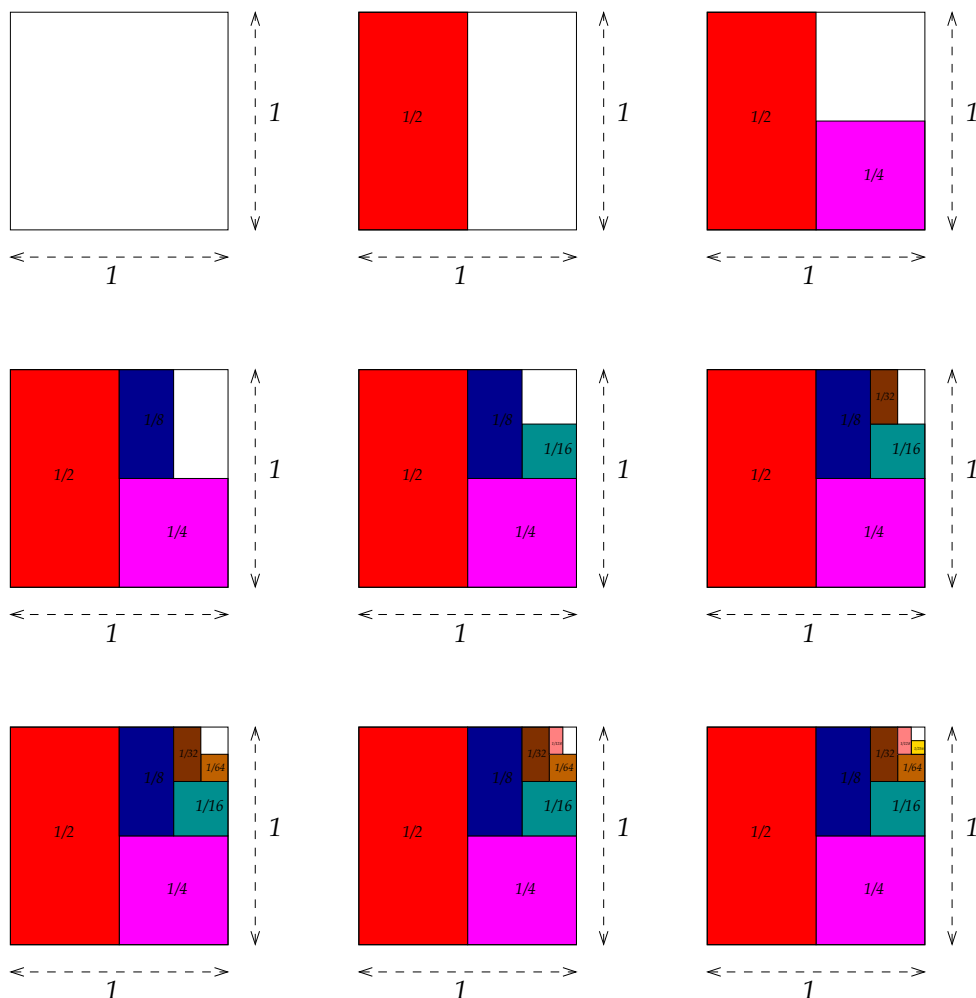
gives infinity, and in such case we say that the series is *divergent*.

It may still happen that summing *infinitely many terms*, one still get a *finite result*! In this case we say that the series is *convergent*.

An example of convergent series:

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \dots = 1.$$

To see it, one can use the following animation:



To make sums efficiently, one often needs to spot *cancellations*.

For example: to sum the infinite series¹

$$\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \dots \quad (1)$$

it is convenient to observe that

$$\begin{aligned} \frac{1}{2} &= 1 - \frac{1}{2}, \\ \frac{1}{6} &= \frac{1}{2} - \frac{1}{3}, \\ \frac{1}{12} &= \frac{1}{3} - \frac{1}{4}, \\ \frac{1}{20} &= \frac{1}{4} - \frac{1}{5}, \\ &\text{etc...} \end{aligned}$$

and therefore, canceling

$$\begin{aligned} -\frac{1}{2} &\text{ with } \frac{1}{2}, \\ -\frac{1}{3} &\text{ with } \frac{1}{3}, \\ -\frac{1}{4} &\text{ with } \frac{1}{4}, \\ -\frac{1}{5} &\text{ with the subsequent } \frac{1}{5}, \end{aligned}$$

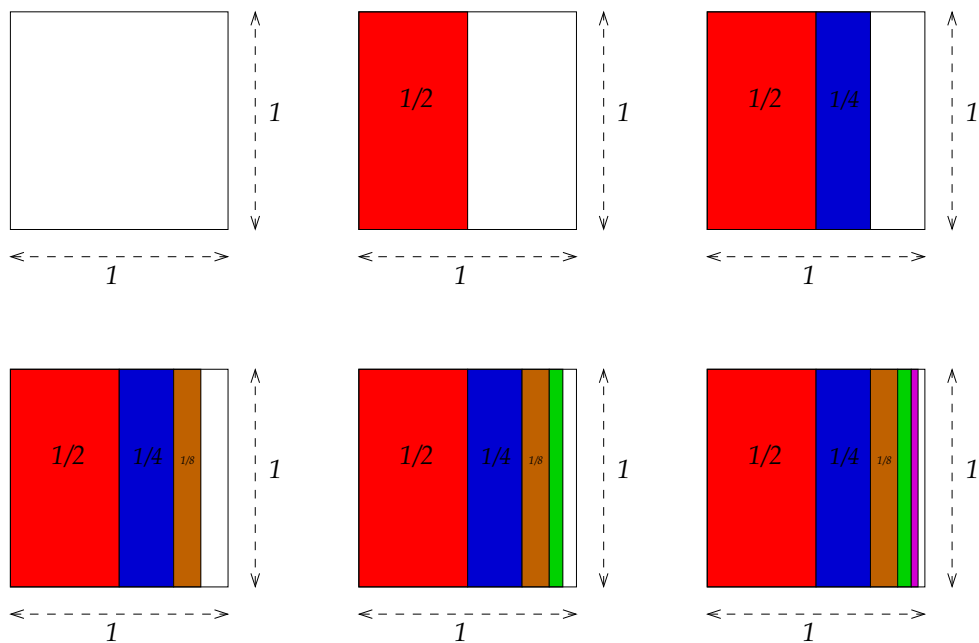
we see that

$$\begin{aligned} &\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \dots \\ &= \left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \left(\frac{1}{4} - \frac{1}{5}\right) + \dots \\ &= 1. \end{aligned}$$

¹The denominators in (1) are obtained by the expression $n(n+1)$, with $n \in \{1, 2, 3, 4, \dots\}$.

ACTIVITY: use the following animation to see that

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \cdots = 1.$$



ADDITIONAL ACTIVITY FOR GIFTED STUDENTS: use the following animation to see that

$$\frac{3}{4} + \frac{3}{16} + \frac{3}{64} + \frac{3}{256} + \cdots = 1.$$

