

Lesson Plan

NEUROHEADWAY

Neuroscience Into Education

Similar Triangles

Module intended for students of Key Stage 2 (7-11 years old).

Learning opportunities include:

- What are similar triangles?
- How to produce similar triangles using parallel lines?
- How to measure the height of tall objects?

1 hour lesson plan

Part	Subject	Description	Time
1	Congruent triangles	Show a set of triangles of different colors, sizes and shapes. Explain congruency. Identify congruent triangles. Cut out paper triangles and check that congruent ones can be superimposed.	10
2	Similar triangles	Show a set of triangles of different colors, sizes and shapes. Explain similarity. Identify similar triangles.	10
3	Intercept theorem	Explain the statement, drawing examples on the blackboard. Place previously cut triangles as in the theorem.	10
4	Measuring heights	Learn how to measure the height of a tall object by using proportionality relations between sides of similar triangles	25
5	Closing remarks	Recall main facts, discuss more complex situations	5

Resources: scissors, paperboard, pin, thread, a small heavy object (e.g., a nut), a tape for measuring distances.

Files provided separately: gt.pdf contains supplementary content for gifted and talented pupils, team.pdf contains supplementary content for teamwork based sessions of study, triangles.

The teacher should provide the supplementary content to gifted and talented pupils, taking care that they do not miss the general presentation, especially the intercept theorem.

Lower ability pupils should work in team with more advanced pupils.

Lesson

Part 1

The teacher invites the students to look at the following picture (or similar one)



We say that two triangles are congruent if corresponding sides have equal lengths and corresponding angles are also equal. Students are asked to identify pairs of congruent triangles. In the above example, the pairs are green and light blue, orange and light gray, pink and magenta.

The teacher gives to each student the above drawing (or similar one) printed. The students are asked to cut out the triangles, using scissors, and check that congruent ones can be superimposed. The teacher stresses that often a rotation is needed and, in some cases, one of the triangles must be flipped over for the superposition to happen (in the above example, flipping is necessary for the green and light blue triangles).

The use of scissors can be avoided if the teacher provides already-cut triangles.

Part 2

We say that two triangles are similar if corresponding sides are in proportion and corresponding angles are equal. One of the triangles is like a zoomed view of the other. Congruent triangles are considered to be similar. Students are asked to identify pairs of similar triangles. In the above example, the pairs are pink and

magenta, dark blue and dark gray, orange and light gray. There is also a triplet of similar triangles: light blue, green and yellow.

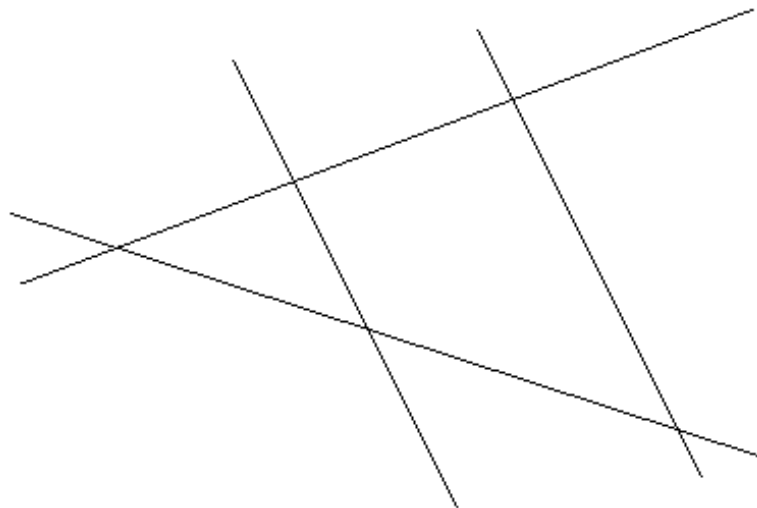
The teacher explains that, for a given pair of triangles, the proportionality factor is a constant. For instance, in the example above, each side of the yellow triangle is half of the corresponding side of the light blue triangle.

Part 3

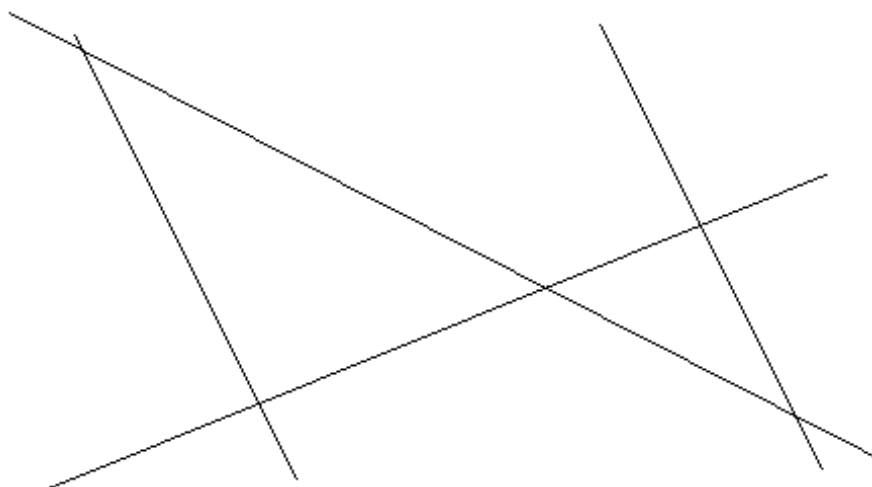
The teacher explains the following statement:

Intercept theorem (also known as Thales' theorem) Two parallel lines cut other two intersecting lines into similar triangles.

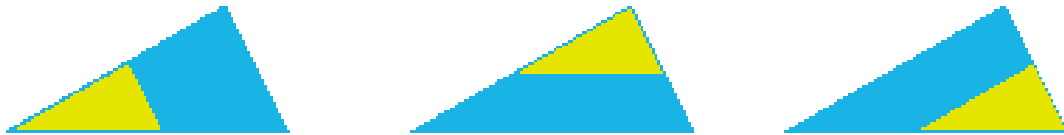
Two cases are to be presented, one when the intersection point is outside the parallel lines:



and another one when the intersection point lies between the two parallel lines :



The teacher shows that any two similar triangles can be brought to a position like in the intercept theorem. For instance, the yellow and light blue triangles can be positioned like this:

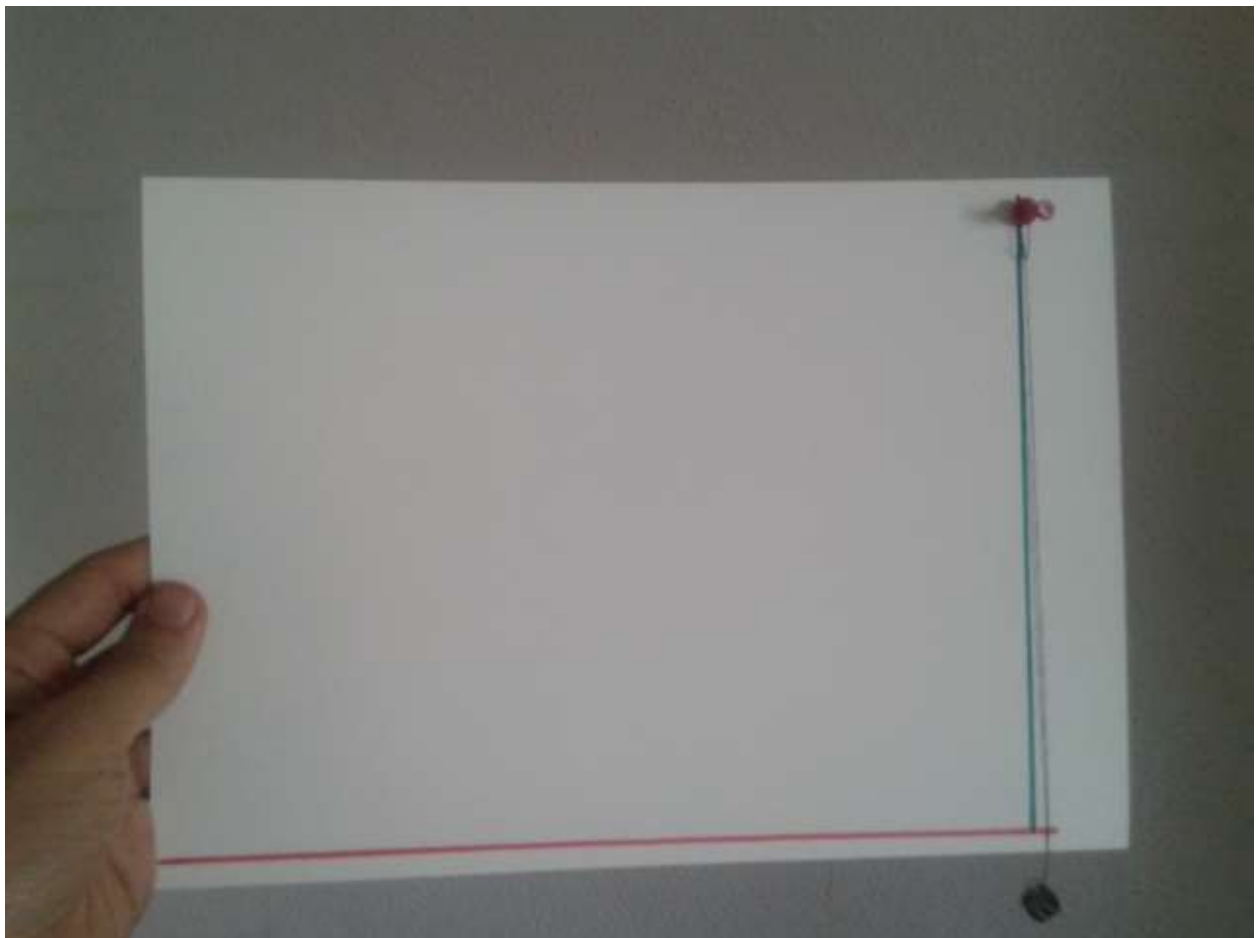


Again, a flip-over may be necessary in some cases.

Part 4

The teacher introduces the question of measuring the height of tall objects, explaining that vertical distances are difficult to measure. For instance, we would like to be able to measure the height of a tree without having to climb it. On the other hand, horizontal distances are easier to measure. Similarity of triangles provides a mean to compute heights by measuring horizontal distances.

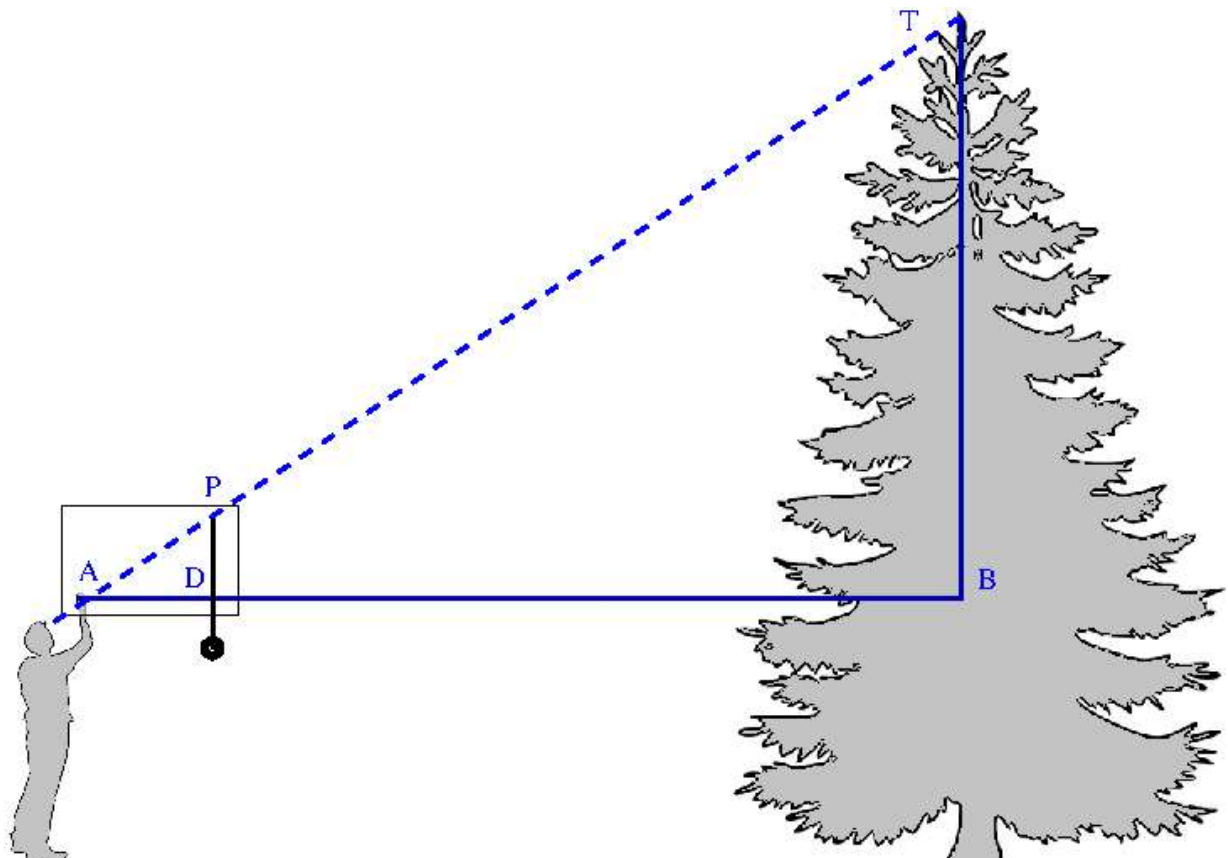
The teacher gives to each pupil paperboard, a pin, thread and a nut and explains how to build a device like this one (pupils may work in teams of two):



It is important that the green and red lines be orthogonal. The thread should be pinned onto the paperboard and should have a small heavy object attached. Its role is to ensure that the green line is vertical, and thus the red line is horizontal.

If the object to be measured is very tall, we might prefer to switch the proportions of the paperboard, making the vertical line longer than the horizontal one.

Here is how this device should be used (outside). We place ourselves in front of a tall object (a tree or a building). We look at the object alongside the device, using the thread to make sure that the green line stays vertical. We align the pin P with the top T of the object and we mark the point A where the viewing direction intersects the horizontal line on the paperboard. Students may work in teams of two. We then measure the horizontal distance $|AB|$ from the place where we are to the bottom B of the tree or building. The final computation can be done in the classroom.



The teacher explains that, since the line DP was kept vertical, two similar triangles appear, ABT and ADP . This means that corresponding sides are in proportion; ABT is like a magnified view of ADP . The first step is to compute the zoom, or the magnifying factor (for instance, the sides of ABT could be twenty times the corresponding sides of ADP , in which case the magnifying factor is 20). We compute

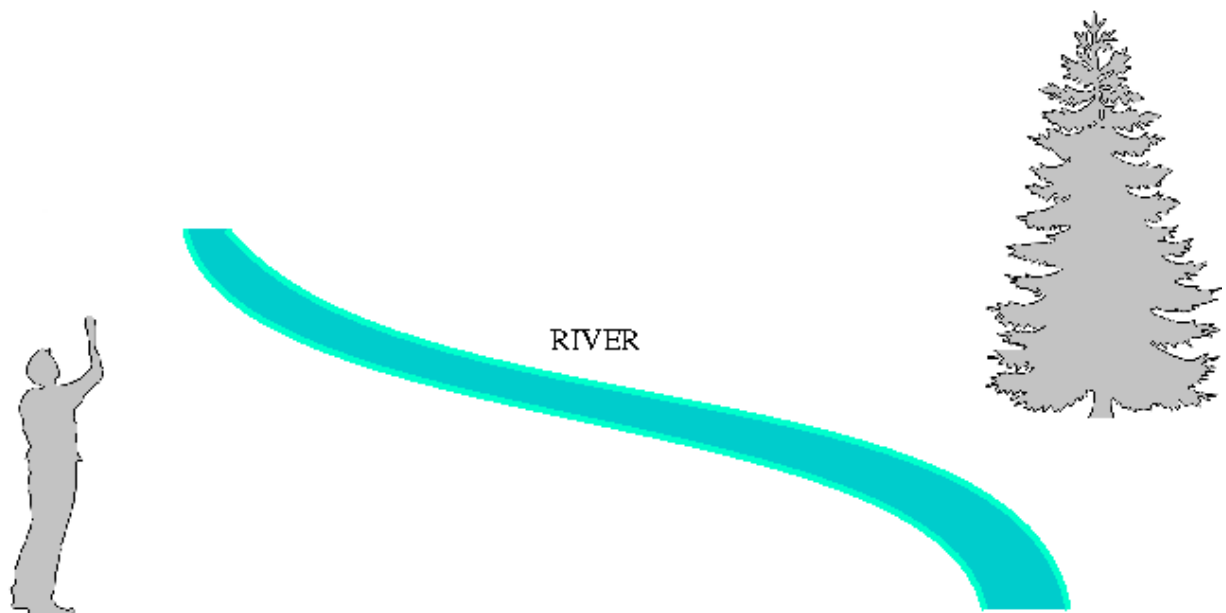
the magnifying factor as the ratio $\frac{|AB|}{|AD|}$. We then compute the desired height $|BT|$ by multiplying $|DP|$ by the magnifying factor.

We have not taken into account the height of the device when held in our hand; we may want to add it.

Part 5

The teacher recalls the main facts. The thread ensures that the green line stays vertical, so two similar triangles show up. In order to compute the magnifying factor, we need to measure the horizontal distance between ourselves and the bottom of the object.

More complicated situations may appear in practice. For instance, the bottom of the object may be inaccessible:



Or, there may be a difference of altitude between the observation point and the object (the computations in this module assume that the land is flat):

